ORIE 5355

Lecture 10: Algorithmic pricing: price differentiation, competition, and practice

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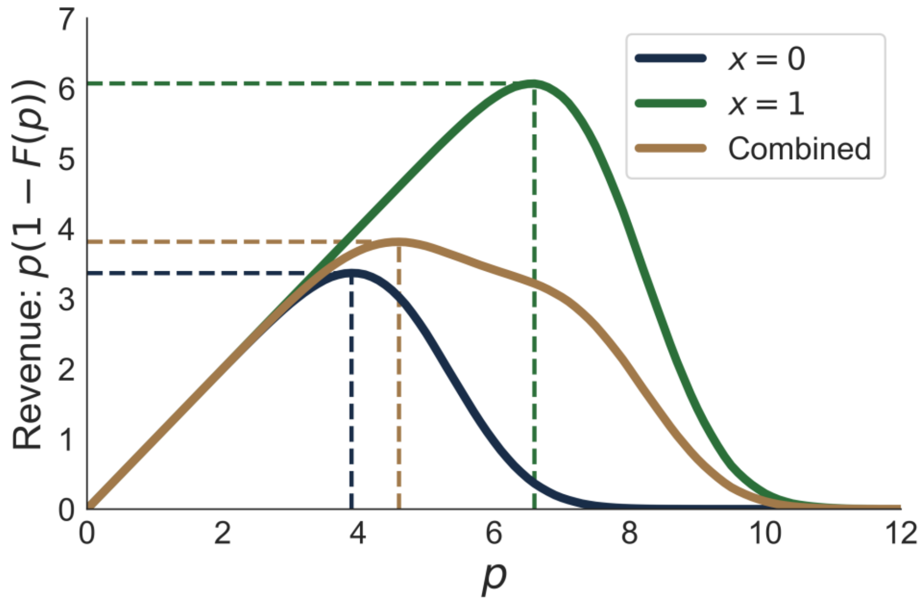
Announcements & reminders

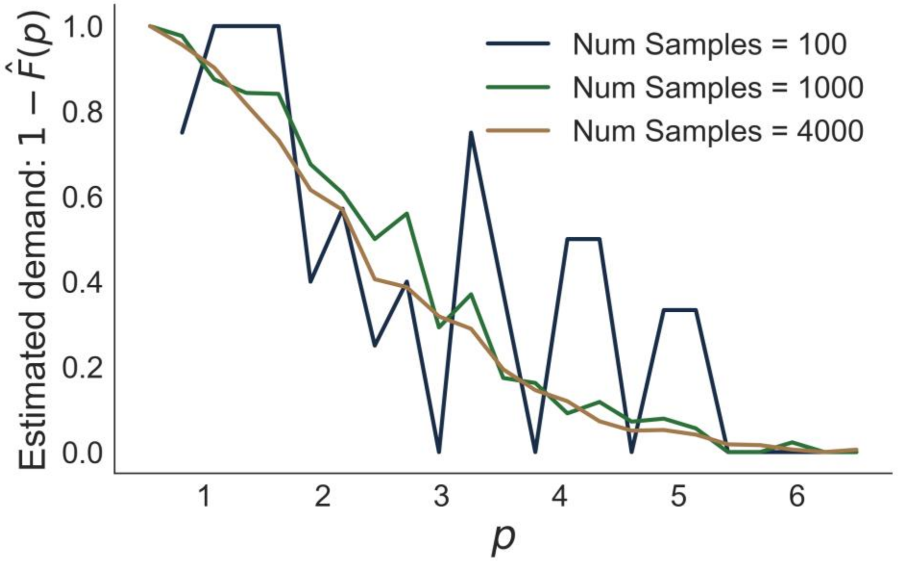
- HW 3 due next week
  - Quiz 3 next week too
- In person pricing ethics discussion next Wednesday! **Important**

Pricing so far

- Given a demand distribution $\mathrm{d(p)} = 1 - F(p)$, how to calculate optimal prices
$$\arg\max_{\mathrm{p}} [\mathrm{p} \times \mathrm{d(p)}]$$

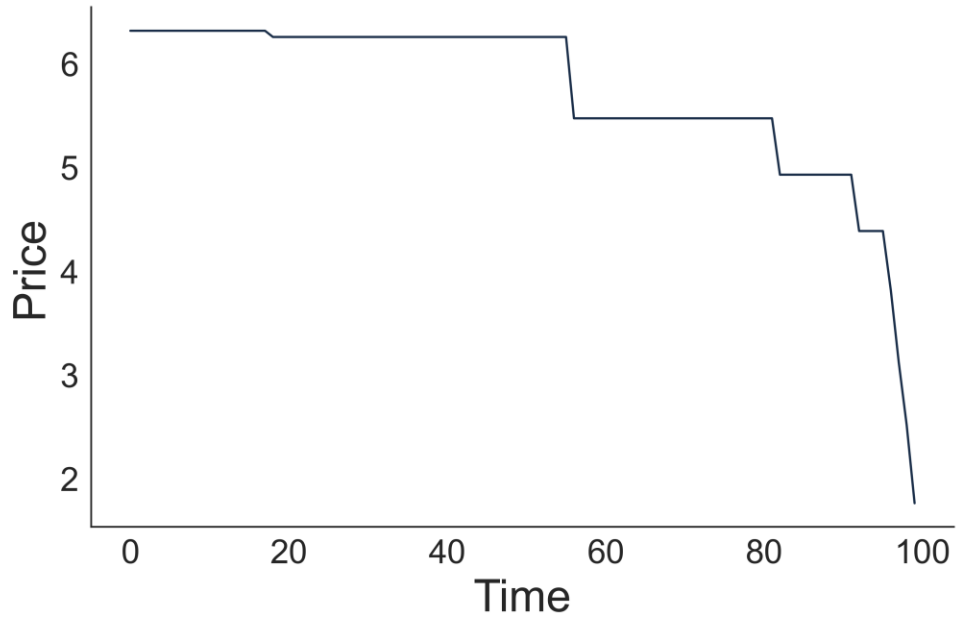
- How to estimate demand distributions, potentially as a function of covariates





Capacity constraints and pricing over time

- Dynamic programming approach
- If you have T time periods to sell an item and want to maximize expected revenue, what prices $p_1 \ldots p_T$ do you set?
- Key idea: optimize backwards
  - First decide price p_T
  - Then decide price p_{T-1}
- Posted additional notes; come to OHs for additional questions



Goal: Maximize expected revenue starting at time $t=0$ ($= V_0$) by setting prices $p_0, \dots p_2$ if have until time $T-1=3$ to sell it

Prob sell item at time 0: $d(p_0)$

revenue if sell at time 0: p_0

revenue at $t=0$: 0

Prob sell item at $t=1$: $d(p_1)$

revenue if sell at $t=1$: p_1

Prob. don't sell item at time 0: $(1-d(p_0))$

$$V_0 = d(p_0)\,p_0 + (1-d(p_0))\Big[0 + d(p_1)\,p_1 + (1-d(p_2))\Big[d(p_2)\,p_2 + (1-d(p_2))\big[d(p_3)\,p + (1-d(p_3))\,0\big]\Big]\Big]$$

$= V_3$: $\big[d(p_3)\,p + (1-d(p_3))\,0\big]$

don't sell at time $t = T-1 = 3$: $(1-d(p_3))$

$V_2 =$ the green bracketed term

$V_1 =$ the red bracketed term

Then: with $T-1=3$ as last day to sell item

$$V_{T-1} = V_3 = d(p_3)p_3 + \cancel{(1-d(p_3))0}$$

$$V_2 = d(p_2)p_2 \quad (1-d(p_2)) V_3$$

$$V_1 = d(p_1)p_1 + (1-d(p_1)) V_2$$

$$V_0 = d(p_0)p_0 + (1-d(p_0)) V_1$$

General equation:

$$V_T = 0$$

for $t=0 \ldots t=T-1$: $V_t = d(p_t)p_t + (1-d(p_t)) V_{t+1}$

Bellman equations: a general idea

- Constructing a tree to reason about what happens tomorrow, and then iterating backwards, is a powerful + flexible algorithmic technique: "dynamic programming"
- Example: What if you have 5 copies of each item?

  Let k denote how many copies of the item I have. Then:

$$V_{t,0} = 0 \text{ for all } t$$

$$V_{t,k} = \max_{p_{t,k}} d(p_{t,k})[p_{t,k} + V_{t+1,k-1}] + (1 - d(p_{t,k}))\, V_{t+1,k}$$

If I sell an item today: Revenue today, plus future revenue from 1 less item

If I don't sell: Future revenue from same number of items

Competing effects: Now, less capacity over time → prices should go up (but less time to sell, so prices should go down).

Capacity constraints + over-time pricing in practice

- Dynamic programs/bellman equations are powerful, but often the real world is too complicated
  - Uncertainty in future capacity
  - Future actions of competitors
  - Future demand distributions
  - "Long time horizons" (T is big)
- In theory, dynamic programming can handle the above. In practice, hard to know how to calculate future value.

Approximating dynamic programming

- In the recommendations module, we created “score”(or “index”) functions:
  - Consider future users, through capacity and avg ratings terms in the score function
- With 1 item: V_{t+1} represents my “opportunity cost” if I sell an item today that I could have sold tomorrow.

  Also interpret as “safety net”: if fail to sell the item today, still earn V_{t+1} in expectation
- Instead of doing a full Bellman equation, estimate V_{t+1} through some other means, then plug into the decision problem for today (finding price p_t)
  - Can construct it like we did score functions for recommendations
  - AlphaGo to play Go: V_{t+1} is partially estimated via a neural network

Pricing with capacity summary

- Just like in recommendations, have to think about potential future demand
- Here, potential future demand lets us be “more aggressive” by pricing higher today
- If I can summarize future revenue (V_{t+1}) effectively, then I can optimize today’s prices
- Dynamic programming: start from the end!
- We assumed that customers can’t strategize on when to come – not true!

Questions?

Plan for rest of today

Last time:

- A little bit on using side-information (user and item vectors) to estimate personalized demand
- Capacity constraints over time

Many assumptions from previous lectures:

- Only one item
- Allowed to explicitly give different prices to different users
  - Or give different prices over time
- No competition from other sellers
- No over-time dynamics

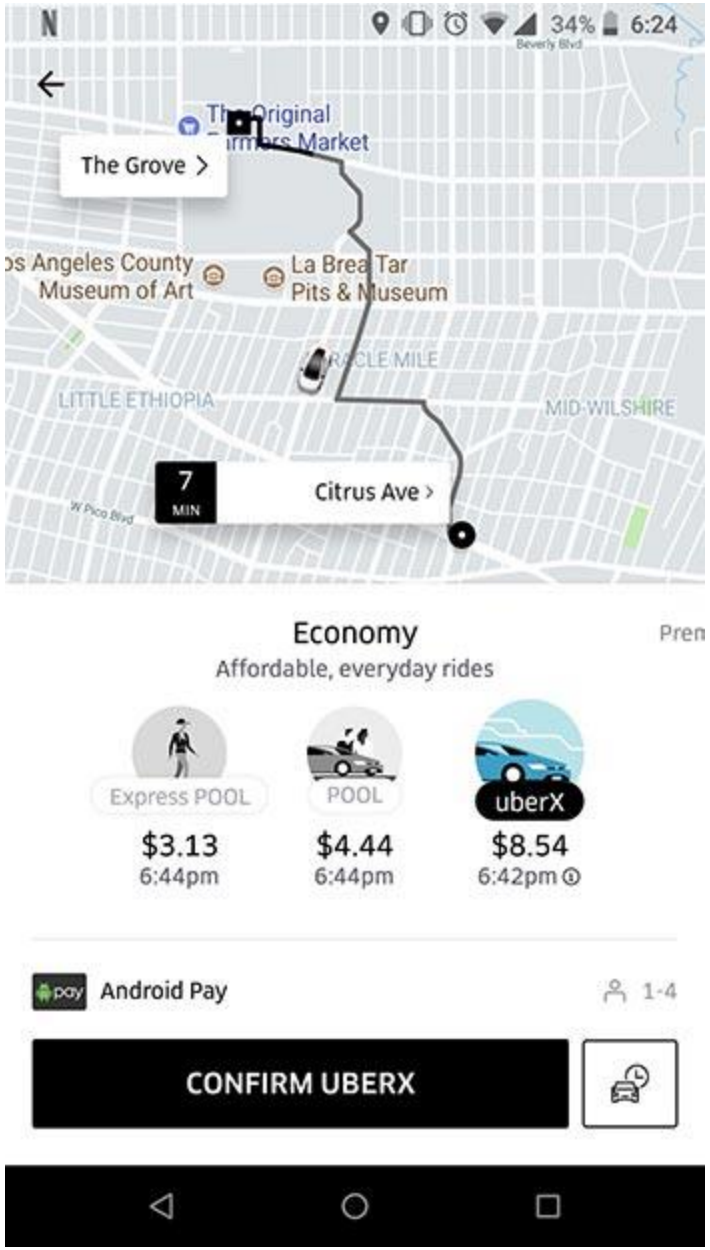
We'll peel back some more of these assumptions today

Selling multiple kinds of items

Price differentiation

Example

- Ride-hailing offers different “tiers” of service
- UberPool cheaper than UberX
  - Also costs less for the platform
- How do we price these items together?
  - What happens if we do simple revenue maximizing price for each item separately?
  - What happens if we make UberPool cheaper?



Motivation

Motivation 1:

You simply have multiple kinds of products to sell. Different types of clothes, laptops, airline seats, furniture, etc.

Motivation 2:

- Earlier: personalized pricing with covariates
- Challenge: Often you can't (technically, ethically, legally, …) give different prices for the same product to different users based on covariates
- Now: Different "tiers" of service.
  - High quality: First class seats, faster service in Uber/Lyft, luxury goods versions, get item "now"
  - Lower quality: Economy seats, UberPool/Lyft Wait and Save, …

=> Purposely create tiers of service to earn more money from richer people while earning something from others

Challenges

- Just like pricing over time, now prices for the 2 items depend on each other
  - Unlike pricing to different demographic segments without capacity constraints

- Cannibalization: Customers who would have bought the luxury good instead buy the cheaper good because it is available

2-item user behavior model

- Suppose you're selling 2 types of items
- Each person will buy at most one item
  - Each person has a *private valuation* v_1 for item 1 and v_2 for item 2
- Suppose you offer the items at price p_1 and p_2, respectively
- How does the person make their decision?

  Utility from item j at price p_j is $v_j - p_j$
- Person i buys

  Neither item if $v_1 < p_1$ and $v_2 < p_2$

  Item 1 if $v_1 \geq p_1$ and $v_1 - p_1 \geq v_2 - p_2$

  Item 2 if $v_2 \geq p_2$ and $v_2 - p_2 \geq v_1 - p_1$

Assumption on customer's "choice model." More generally, customer could buy randomly, with choice probabilities that depend on

$$v_j - p_j$$

In more detail

How does the person make their decision? Person i buys

Neither item if $v_1 < p_1$ and $v_2 < p_2$

Item 1 if $v_1 \geq p_1$ and $v_1 - p_1 \geq v_2 - p_2$

Item 2 if $v_2 \geq p_2$ and $v_2 - p_2 \geq v_1 - p_1$

Revenue in 2 item model

For a set of prices (p_1, p_2), let

$\mathrm{d}_1(p_1, p_2)$ be fraction of people who buy item 1

(Yellow Region)

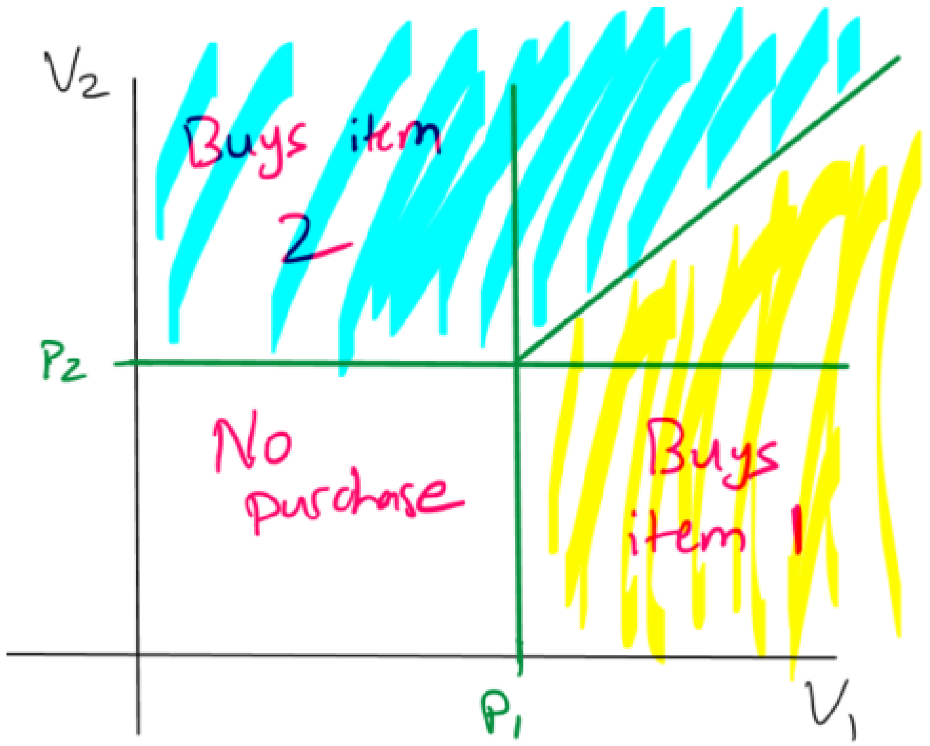
$\mathrm{d}_2(p_1, p_2)$ be fraction of people who buy item 2

(Blue Region)

Then, revenue is:

$$p_1 \times \mathrm{d}_1(p_1, p_2) + p_2 \times \mathrm{d}_2(p_1, p_2)$$

Given functions $\mathrm{d}_1, \mathrm{d}_2$, can solve for optimal prices



Cannibalization

Now, each price affects other item.

Revenue: $p_1 \times \mathrm{d}_1(p_1, p_2) + \ p_2 \times \mathrm{d}_2(p_1, p_2)$

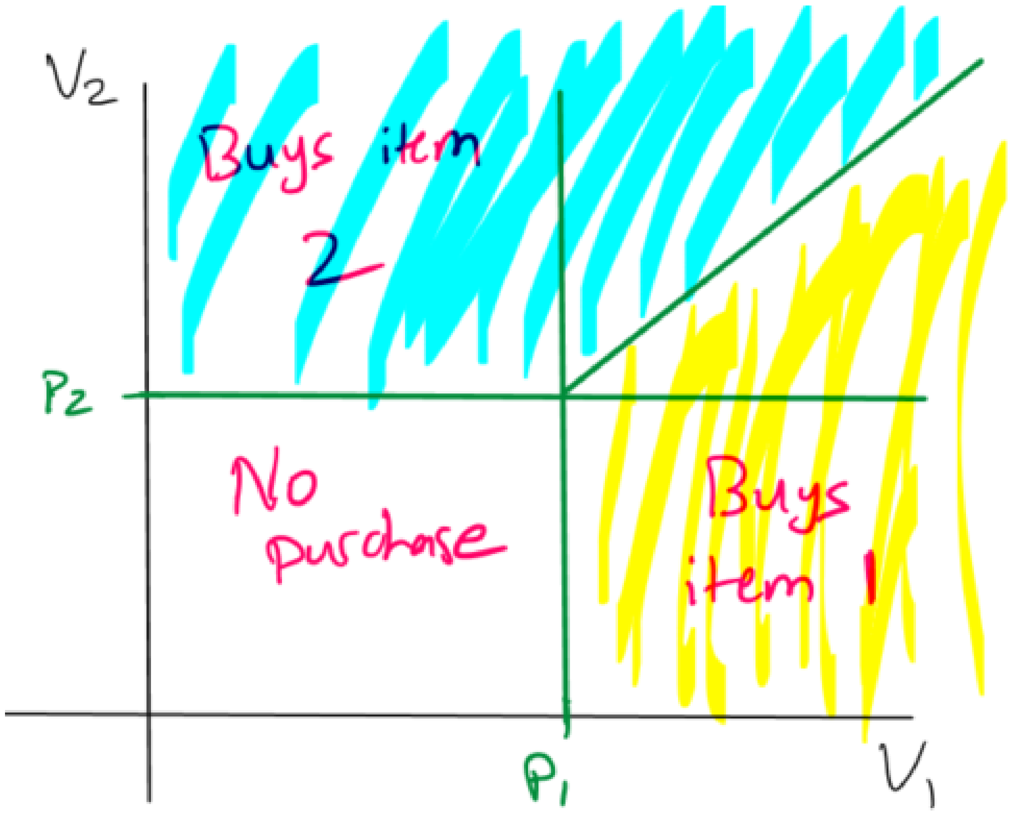
Suppose decrease p_1 (make item 1 cheaper)

Then:

- Earn less money in yellow region ↓
- Yellow region becomes bigger

  White region becomes smaller ↑

  Blue region becomes smaller ↓



Demand estimation with multiple items

- With a single item, we suggested machine learning approach to estimate: $\mathrm{d}(\mathrm{p}, \mathrm{x}) \stackrel{\text{def}}{=} 1 - F_{p|X}\,(p \mid X = x)$
- Assume we have user i with covariates x_i
- Now, would need to estimate $\mathrm{d}_1(p_1, p_2, x_i)$ and $\mathrm{d}_2(p_1, p_2, x_i)$
- Gets very hard, very quickly
- Approach 1: Use a *multi-class* classification algorithm $g(p_1, p_2, x_i)$

  [Buy nothing, buy item 1, buy item 2] and then extract class probabilities

  (sci-kit learn: use **predict_proba** with any multi-class classifier)
- Approach 2: (Extend idea from previous class)
  - Use user and item vectors, i.e., $(\mathrm{p}_1, \mathrm{p}_2, u_i \cdot w_{\text{item 1}}, u_i \cdot w_{\text{item 2}})$

Sidenote: Substitutes and complements

- So far: motivation -- we have multiple products to sell, that appeal to different customers

  "cheaper" and "more expensive" product
- Items are "substitutes": people only buy at most one kind of item
- Sometimes, items are "complements" – buying one item makes the other item more attractive
  - Soda + popcorn at movie theater
  - iPhone and Macbook and Apple Watch and Apple TV and …
- Then, reducing one item's price might induce you to buy more overall
  - An item is a "loss leader"

Putting pieces together: class competition

So far we’ve covered

- Recommendation systems
  - Given past user and item data, predicting how much each user would like each item
  - How to turn these predictions into *recommendations* (with capacity constraints)
- Pricing
  - Single item revenue maximization
  - Estimating demand at each price, potentially with covariates
    - Potentially with multiple items, and with using user and item vectors
  - Pricing over time with capacity constraints
  - Pricing multiple items

Overview: Real-life algorithmic pricing

- You and a single competitor (your classmates) each are selling two types of items, Book A and Book B.
  - (Potentially: suppose you get K copies of each item every 10 steps)
- A customer walks in and you observe some personal data
  - Just demographic covariates
  - Demographic covariates & user vector trained using their past experiences
- You and your competitor post prices for each item
- The customer at most 1 item and leaves
- Repeat for many customers over time

Basic case

- For now, let's ignore competition
- For each user, you have either just demographic covariates x_i or also a trained user vector u_i from their past interactions on your site
- You would predict demand for each item, $d_1(p_A, p_B\ x_i, u_i)$ and $d_2(p_A, p_B, x_i, u_i)$ for each set of prices (p_A, p_B)
  - Your choice on how to estimate this demand
  - What do you do for customers with no user vector u_i?
- Set prices to maximize your expected revenue

Complication 1: Capacity constraints

- Now, have K copies of each item for each T=10 customers.
- Now, the price that you set for each item should depend on opportunity cost: what if you can sell that item to a different customer in the future?
- 3-d Bellman equation: time, capacity of Book A, capacity of Book B
- Set up your Bellman equation:

$$V_{t,k_A,k_B} = A + B + C$$

A: If I sell Book A today: Revenue today, plus future revenue from 1 less Book A

B: If I sell Book B today : Revenue today, plus future revenue from 1 less Book B

C: If I don’t sell anything: future revenue from same number of copies

How to calculate future revenue?

- As before, future revenue depends on future prices that you set
- …Think about prices you'd set on last day T-1=10
  - For each combination of capacities left $\mathrm{k}_A, \mathrm{k}_B$
- Complication: on day $\mathrm{t} < \mathrm{T} - 1$ you don't yet know the customer x_{T-1}, u_{T-1} that will show up on the last day $\mathrm{T} - 1$!
  - You only know customer who has shown up on day t
- When calculating future *expected* value $\mathrm{V}_{\mathrm{t}+1,\mathrm{k}_A,\mathrm{k}_B}$, you need to consider the *distribution* of customers that *could* show up
  - Use training data to consider possible customers that could show up
  - Then calculate the prices that you *would* show each of them

Complication: Competition

- You and your opponent both do the same thing, and calculate the exact same prices p_A, p_B at the current time step
- Your opponent is clever, and so decides to *undercut* you slightly, and so sets prices $p_A - \$0.01, p_B - \$0.01$
- …but you're cleverer, and know your opponent will do this, and so you set prices $p_A - \$0.02, p_B - \$0.02$
- There's now a game theory component: you need to anticipate what your opponent will do when setting prices
- More complicated: it's a repeated setting
  - You can *learn parameters* for how your opponent behaves

Rest of pricing module

Monday: Pricing in ride-hailing [+ congestion pricing]

Wednesday: What’s acceptable in pricing? (In person, will take attendance)

Questions?