ORIE 5355: Applied Data Science - Decision-making beyond Prediction

Lecture 3: Survey weighting methods

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Announcements

- Homework 1 posted
- Please make sure you have access to EdStem and are receiving announcement notifications
- Office hours set: (all on course calendar) Location TBD
- Contact each other via the homework buddy form
- Wait lists should be clearing soon: contact Student Affairs (not me)

Mean estimation from surveys

The task: estimate mean opinion

- Each person j has an opinion, $Y_j \in \{0, 1\}$
- We want to measure $\bar{y} = E[Y_j]$, the population mean opinion on some issue
- Each person also has covariates, x_j^k
- We also may care about *conditional* means

$$E[Y_j \mid \text{ORIE program}]$$

Example:

"Do you like the class so far?"

Options: "yes" and "no"

$\bar{y}$: What fraction of people like the class so far?

Degree program, whether you like waking up at 9:30, etc

Fraction of people in ORIE who like the class

This problem is everywhere

- What fraction will vote for the Democrat in the next election
- What is the average rating of this product?
- Do people want the city to build a foot bridge to Manhattan?
- Are people happy with this new feature I just deployed?

Naïve method: take the mean

- Get list of people (watched the movie; from phone book)
- Call them, suppose everyone answers and get Y_j from each
- We now have $\{Y_j\}_{j=1}^{N}$, if called N people

Random sample of people in this class

- Simply do, $\hat{y} = \frac{1}{N}\sum_j Y_j$

Average opinion of the sample

- By law of large numbers, if Y_i is independent and identically distributed according to the true population's opinion, then

$$\hat{y} \rightarrow \bar{y} \text{ as } \mathrm{N} \rightarrow \infty$$

$\bar{y}$: Actual opinion of the class

What goes wrong

People don’t give “true” opinion

Why?

- You’re asking about something sensitive
- “social desirability” – people like making other people happy
- They’re getting paid to answer the survey and just want to finish
- You know they other person is also going to rate you

Of course, then you’re (likely) not going to succeed

People gave you $\widetilde{Y}_j$, instead of Y_j

$\hat{y} = \frac{1}{N}\sum_j \widetilde{Y}_j$

You lie because you want a better grade

$\hat{y}$ does not converge to $\bar{y}$, *unless errors cancel out*

Your sample does not represent your population

- You just posted a poll on Facebook or Twitter, anyone could respond
- You called only landlines, and no one under 50 owns one anymore
- You only asked people to rate a movie after they've seen it
- You can only rate an item if you bought it *and didn't return it*
- Those with certain opinions are more likely not to answer
    - After bad experiences on online platforms
    - "Shy Trump voters" (?)

=> People who answer the poll are different than your population – "differential non-response"

Your sample does not represent your population, in math

- For each person j, let $A_j \in \{0,1\}$ be whether they answered
- You have $\boldsymbol{Y} = \left\{(A_j, Y_j)\right\}_{j=1}^{N}$, if called N people

  Where $Y_j = \emptyset$ if $A_j = 0$ (they did not answer)
- Again, you do

$$\hat{y} = \frac{1}{|\{j \mid A_j = 1\}|} \sum_{j \in \{j \mid A_j = 1\}} Y_j$$

where $\{j \mid A_j = 1\}$ denotes the set of people who answered

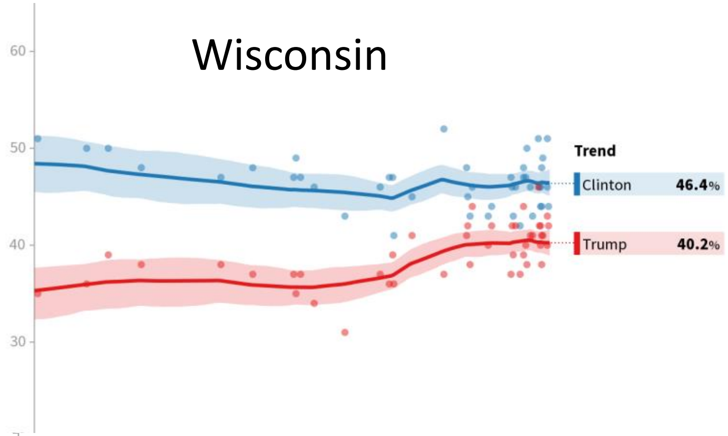
and so $|\{j \mid A_j = 1\}|$ is the number of people who answered

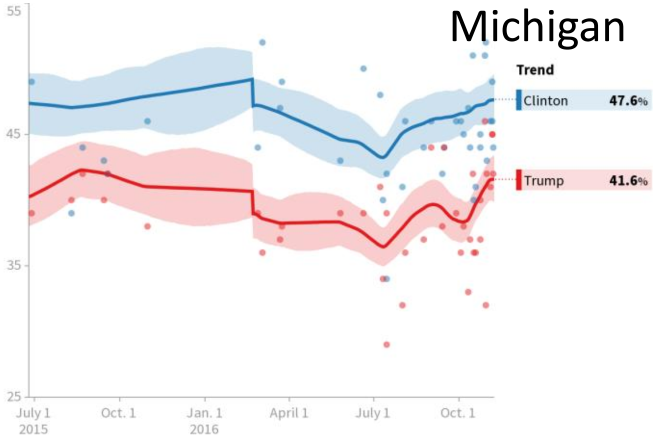
$\hat{y}$ does not converge to $\bar{y}$ *unless* Y_j and A_j are uncorrelated

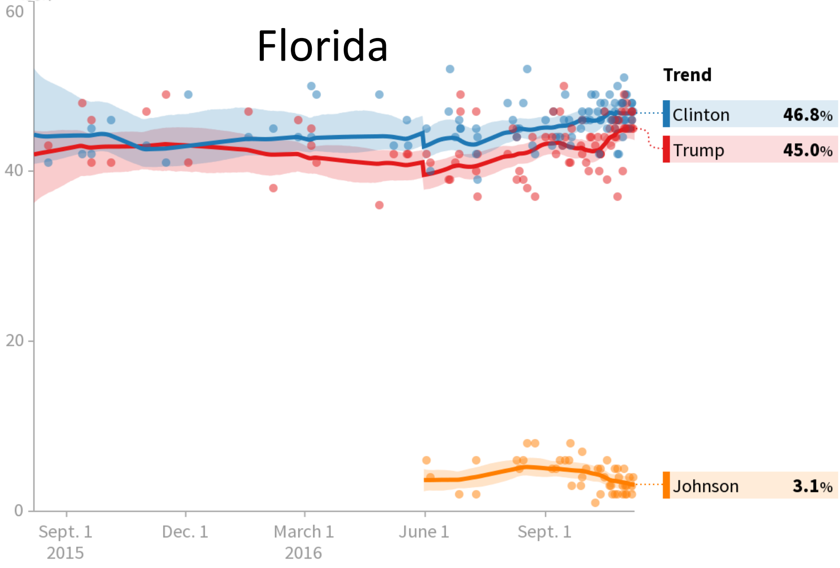
Uncorrelated: Whether you answered is unrelated to what your true opinion is

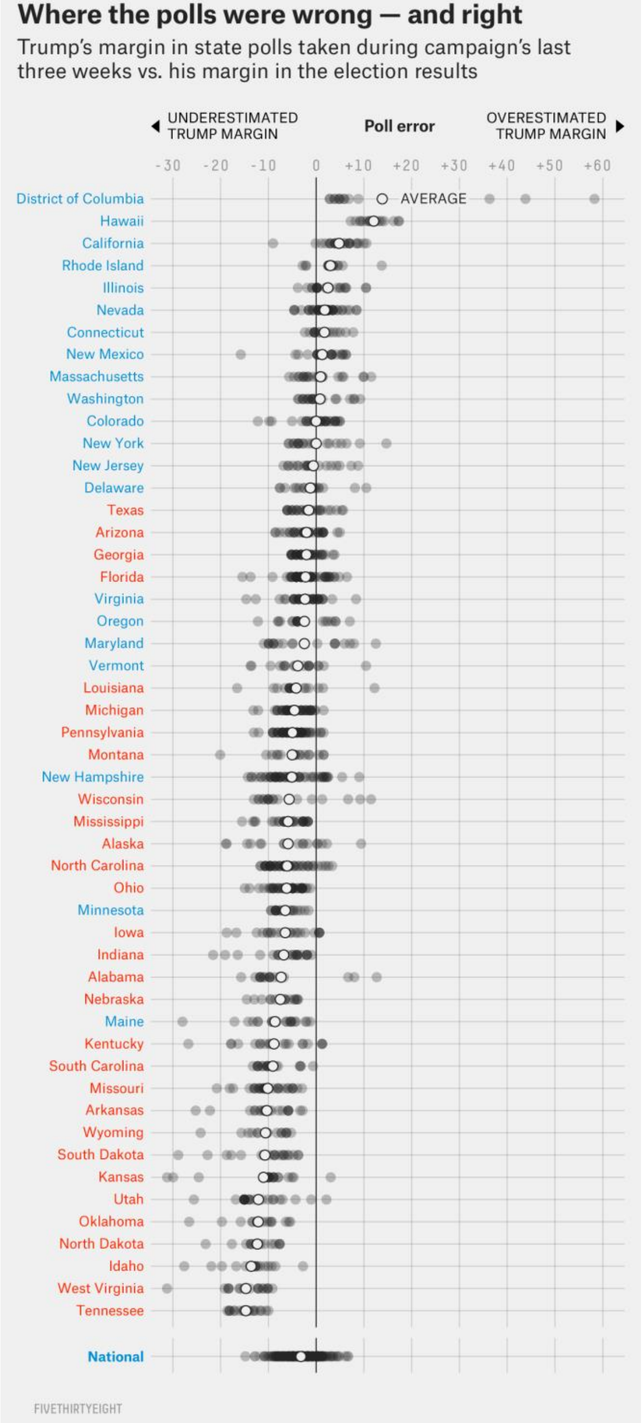
Case study: Polling in US 2016 presidential election

Polls were off (a bit) in the 2016 e







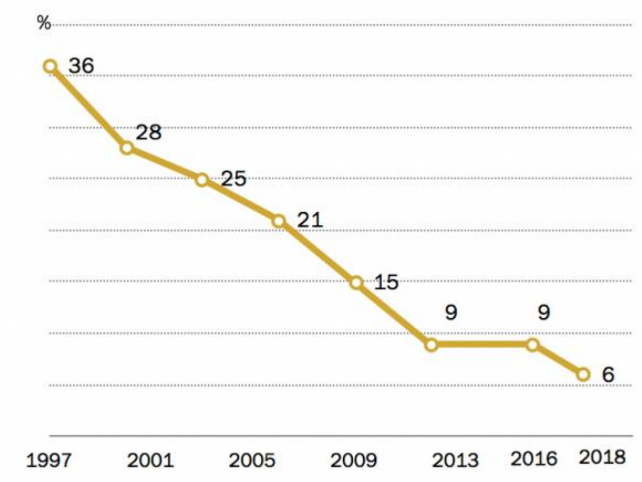


What happened?

- Professional pollsters spend a lot of time on getting opinions right
  - [We'll cover some of their techniques next time]
- But, polling is an increasingly challenging business
  - Basically no one answers a phone poll
  - Modeling opinions/turnout in diverse democracy is hard
  - "social desirability" → "shy Trump voters" (?)
- In 2016, turns out that less educated voters both:
  - Were less likely to answer polls
  - Were more likely to vote Trump

After brief plateau, telephone survey response rates have fallen again

Response rate by year (%)

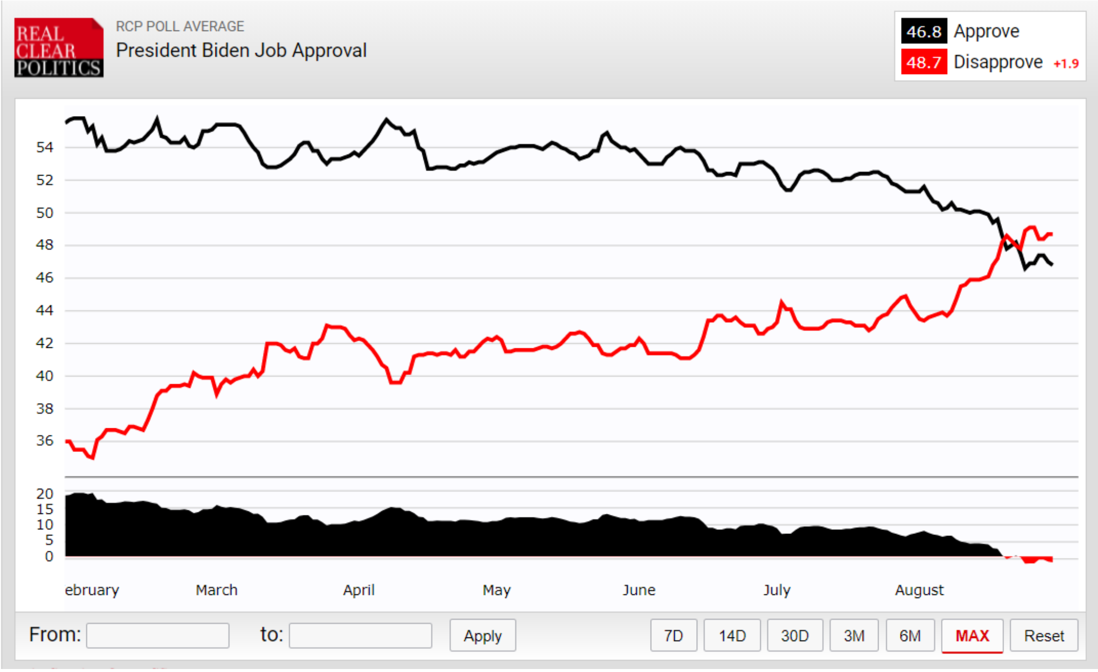


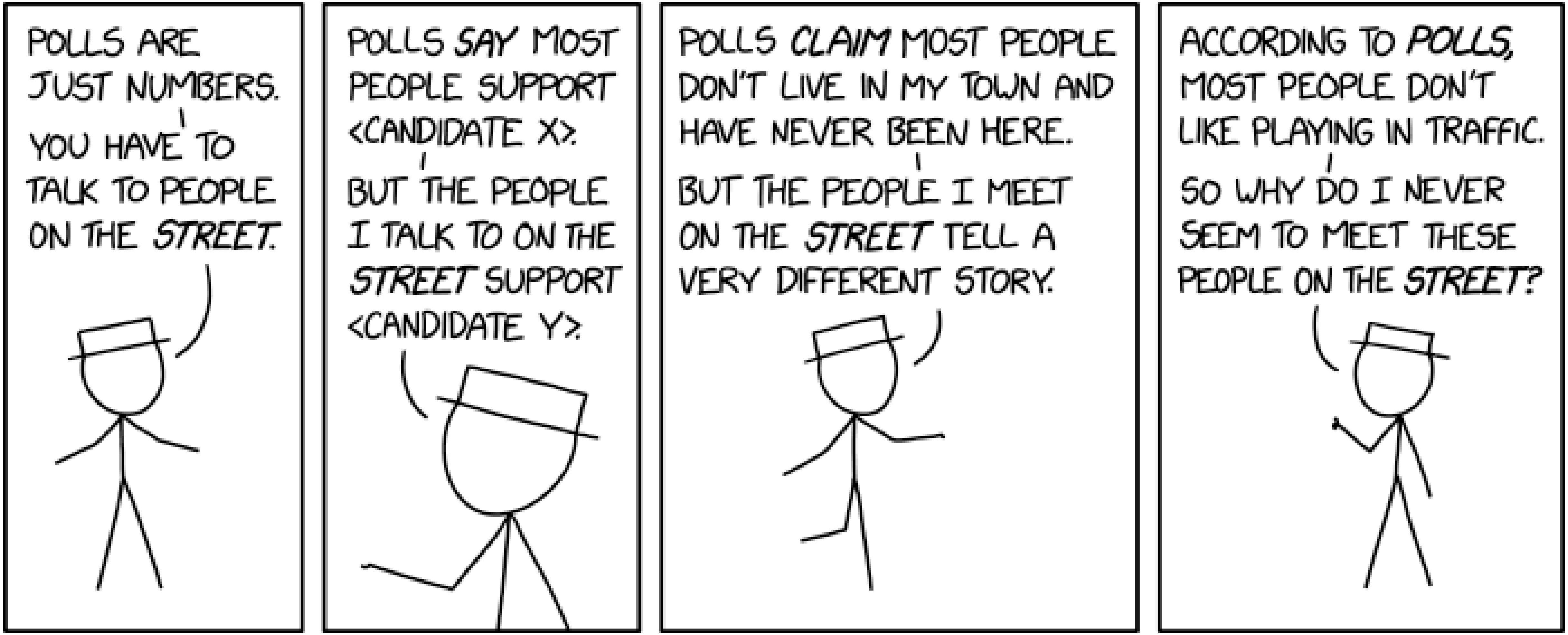
Note: Response rate is AAPOR RR3. Only landlines sampled 1997-2006. Rates are typical for surveys conducted in each year.
Source: Pew Research Center telephone surveys conducted 1997-2018.

PEW RESEARCH CENTER

Differential non-response is everything

- Differential non-response shows up everywhere you're gathering opinions
- Your training data for whatever model you train faces the same issue!
- Standard "margin of error" calculations do not take this into account
- Differential non-response *over time* often explains "swings" in polls!





Other pollsters complain about declining response rates, but our poll showed that 96% of respondents would be 'somewhat likely' or 'very likely' to agree to answer a series of questions for a survey.

[xkcd: Polls vs the Street]

Reminder: the task

- Each person j has an opinion, $Y_j \in \{0, 1\}$
- We want to measure $\bar{y} = E[Y_j]$, the population mean opinion on some issue
- Each person also has covariates, x_j^k
- We also may care about *conditional* means

$$E[Y_j \,|\mathrm{ORIE\ program}]$$

Example:

"Do you like the class so far?"

Options: "yes" and "no"

$\bar{y}$: What fraction of people like the class so far?

Degree program, whether you like waking up at 9:30, etc

Fraction of people in ORIE who like the class

Challenge 1: people don't give "true" opinion

People gave you $\widetilde{Y_j}$, instead of Y_j

You lie because you want a better grade

$$\hat{y} = \frac{1}{N} \sum_j \widetilde{Y_j}$$

$\hat{y}$ does not converge to $\bar{y}$, *unless errors cancel out*

Challenge 2: Sample doesn't represent pop

- For each person j, let $A_j \in \{0,1\}$ be whether they answered
- You have $\boldsymbol{Y} = \{(A_j, Y_j)\}_{j=1}^{N}$, if called N people

  Where $Y_j = \emptyset$ if $A_j = 0$ (they did not answer)
- Again, you do

$$\hat{y} = \frac{1}{|\{j \mid A_j = 1\}|} \sum_{j \in \{j \mid A_j = 1\}} Y_j$$

where $\{j \mid A_j = 1\}$ denotes the set of people who answered
and so $|\{j \mid A_j = 1\}|$ is the number of people who answered

Some people don't answer when asked

$\hat{y}$ does not converge to $\bar{y}$ *unless* Y_j and A_j are uncorrelated

Plan for rest of the day

Methods for tackle sample representation issues

- Stratifying sample *before* you poll
- Weighting techniques *after* you have responses

Differential response on *known* covariates

- Suppose we have a single binary covariate $x_j \in \{0,1\}$ indicating whether they graduated to college
  - Half the population went to college

Whether MEng or MS degree

- Suppose whether people answer is correlated with education

$$\Pr(A_j = 1) = \begin{cases} 0.1 \text{ if } x_j = 0 \\ 0.4 \text{ if } x_j = 1 \end{cases}$$

Whether you answer is correlated with degree program

- Education also correlated with opinion Y_j in some unknown manner
- We want to measure $\bar{y} = E\left[Y_j\right]$, the population mean
- *No other correlations between whether they answer and opinion:*
  - Opinion Y_j *is independent of whether they respond* A_j, *conditional on* x_j

Given your degree program, whether you respond is uncorrelated with your opinion

New notation

- Number of people *called*: N — # of people in class
- Population response rate for group ℓ: A^ℓ — Response fraction in degree ℓ
- Population mean response for group ℓ: $\bar{y}^\ell$ — "Likes class" fraction in degree ℓ
- Population fraction for group ℓ: P^ℓ — Fraction of class in degree ℓ
- Corresponding *sample* values are: $\hat{A}^\ell, \hat{y}^\ell, \hat{P}^\ell$

(i.e., $\mathrm{N}\hat{P}^\ell\hat{A}^\ell = \left|\{j \mid A_j = 1, x_j \text{ in Group } \ell\}\right|$) — # who answered in degree ℓ

and so:

$$\bar{y} = \frac{P^0\bar{y}^0 + P^1\bar{y}^1}{P^0 + P^1} = P^0\bar{y}^0 + P^1\bar{y}^1 = 0.5\,\bar{y}^0 + 0.5\,\bar{y}^1$$

in example — True mean opinion

$$\hat{y}_{naive} = \frac{\hat{A}^0\hat{P}^0\hat{y}^0 + \hat{A}^1\hat{P}^1\hat{y}^1}{\hat{A}^0\hat{P}^0 + \hat{A}^1\hat{P}^1} \rightarrow \frac{A^0P^0\bar{y}^0 + A^1P^1\bar{y}^1}{A^0P^0 + A^1P^1} = 0.2\,\bar{y}^0 + 0.8\,\bar{y}^1$$

Mean poll response

Naïve method in more detail

$$\hat{y}_{naive} = \frac{\left(\sum_{j\in\{j \mid A_j=1, x=0\}} Y_j + \sum_{j\in\{j \mid A_j=1, x=1\}} Y_j\right)}{|\{j \mid A_j = 1,\ x = 0\}| + |\{j \mid A_j = 1,\ x = 1\}|}$$

$$= \frac{\hat{A}^0\hat{P}^0\hat{y}^0+\hat{A}^1\hat{P}^1\hat{y}^1}{\hat{A}^0\hat{P}^0+\hat{A}^1\hat{P}^1} = \frac{(\#(Y_j=1) \text{ from Group } 0\ + \#(Y_j=1) \text{from Group } 1)}{\text{Total Respondants}}$$

$$\rightarrow \frac{P^0A^0\bar{y}^0+P^1A^1\bar{y}^1}{P^0\mathrm{A}^0+P^1A^1} \neq \bar{y} \text{ unless } \mathrm{A}^0 = A^1$$

$P^0A^0/\,(P^0\mathrm{A}^0+P^1A^1)$ is limit fraction of respondents from Group 0

Bias (even with $N \rightarrow \infty$*):* Limit fraction does not match the population fraction

Variance (with finite N*):* Sample values do not match limit values

Stratified sampling

Stratification: change who you call

- Suppose you have L mutually exclusive demographic groups:

  A population that is heterogeneous *across* groups

  Relatively homogenous *within* groups

  (Exactly the setup we have)

Y_j is independent of A_j, conditional on x_j

- Then, instead of calling N completely random people

  Call N^{ℓ} people from group ℓ

  Where N^{ℓ} is determined by how likely each group is to respond

  If MEng students are less likely to respond, call more of them

- Even if each group responds at same frequency, this leads to *lower variance* estimates
- With differential response rates, can also correct the *bias* in *mean*

Why does it work?

- With differential response rate: we can “cancel out” the differential response rate by just calling more people from that group

- Even without differential response rates, just differential opinion:
  There are two sources of variance in estimation:
    Which groups are over- and under- sampled due to noise
    What the opinion of each person is
  Stratification mitigates the first source of variance

Why does it work? (Mathematically)

$$\hat{y} = \frac{\left(\sum_{j\in\{j \mid A_j=1, x=0\}} Y_j + \sum_{j\in\{j \mid A_j=1, x=1\}} Y_j\right)}{|\{j \mid A_j = 1,\ x = 0\}| + |\{j \mid A_j = 1,\ x = 1\}|}$$

$$= \frac{(\#1 \text{ from group } 0\ \ +\#1 \text{ from group } 1)}{\text{Total Respondants}}$$

$$\rightarrow \frac{N^0 A^0 \bar{y}^0 + N^1 A^1 \bar{y}^1}{N^0 \mathrm{A}^0 + N^1 A^1} = \bar{y} \text{ if } \frac{N^0 \mathrm{A}^0}{P^0} = \frac{N^1 \mathrm{A}^1}{P^1}$$

Now
$N^\ell \hat{A}^\ell$ instead of
$\mathrm{N}\hat{P}^\ell \hat{A}^\ell$

Calling more in ratio of non-response

With stratification, cancel out the bias *because* you simply asked more people from the group with lower response rate

It also reduces variance, even if $\mathrm{A}^0 = A^1$ (and $\mathrm{N}^0 = N^1$)

Stratification in practice

- You often don't know group specific response rates A^{ℓ}
  - Define groups and then keep sampling until you have enough samples
  - Weighting after sampling (covered next)
- How many groups/what groups do you choose?
  - Our example had a binary covariate we called "education"
  - What about stratifying ethnicity, or intersectional groups (ethnicity x gender)?
  - Why stop there? Why not ethnicity x gender x education x age …?
  - As number of groups increase, number of people in each group goes down
- Remember the rule: create groups such that the response rates is not correlated with whether they answer, *within each group*

Response Y_j *is independent of whether they respond* A_j, *within each group* x_j

Questions?

Weighting

Main idea for weighting

- In stratified sampling, we balanced out the groups according to their population percentage *before* we called people
- With weighting, we try to do the same thing, but *after* we call people and know how many from each group responded
- Why?
  - You might not know response rates per group
  - You might not know a person's demographics until you call them
  - Can run *sensitivity analyses:* "what would the estimate be if this demographic group only composes x% of the population instead of y%?"
- Comes at a cost: doesn't have the same variance reduction properties as does stratified sampling

Main idea, 2 steps:

Step 1: Use the responses to estimate the mean response for each group ℓ, i.e., get an estimate $\hat{y}^{\ell}$ of the true opinion $\bar{y}^{\ell}$

Step 2: Do a weighted average of $\hat{y}^{\ell}$; each group is given weight W^{ℓ}

$$\hat{y} = \sum_{\ell} W^{\ell}\, \hat{y}^{\ell}$$

If $W^{\ell} = P^{\ell}$ and $\hat{y}^{\ell} \to \bar{y}^{\ell}$, then $\hat{y} \to \bar{y}$

Details differ in how to construct estimate $\hat{y}^{\ell}$, how to calculate weight W^{ℓ}, and what groups ℓ to consider

Naïve Weighting

Step 1: Use the mean response for each group ℓ separately, i.e.

$$\hat{y}^{\ell} = \frac{\sum_{j \in \{j \mid A_j = 1, x = \ell\}} Y_j}{|\{j \mid A_j = 1,\ x = \ell\}|}$$

Step 2: Weight W^{ℓ} is our best guess of true population fraction P^{ℓ} for group ℓ

Complication: How many groups/which ones?

- If group too broad (e.g., group ℓ just gender), then break cardinal rule:

  Need: Opinion Y_j *is independent of whether they respond* A_j, *conditional on group* ℓ

- If group is too specific (*ethnicity* x *gender* x *education* x *age*), then:

  Problem 1: Estimate $\hat{y}^{\ell} = \frac{\sum_{j \in \{j \mid A_j = 1, x = \ell\}} Y_j}{|\{j \mid A_j = 1, x = \ell\}|}$ might be really bad

  Too few respondents in a group → high variance (1 person might determine entire average)

  Problem 2: We might not know population fraction P^{ℓ}

Tackling Problem 2: Population weights

- Suppose very specific group (*ethnicity* x *gender* x *education* x *age*)
- Naïve: try to figure out true population fraction ("joint distribution")

  "W^{ℓ} = P^{ℓ} *fraction of pop is college educated white women age 35-44*"

- Easier: Use "marginal" distribution for each covariate

  "a fraction of population is women"

  "b fraction of population is college educated"

  "c fraction of population is white"

  "d fraction of population is age 35-44"

  $\Rightarrow$Pretend "W^{ℓ} = *abcd fraction of pop is college educated white women age 35-44*"

- *Not covered -- "raking": match marginal distribution for each covariate without assuming that marginal distributions make up joint distribution*

The homework

- In the homework, first we define groups just based on a single covariate, for example gender, ethnicity/race, political party, etc.
  - (e.g., group ℓ just based on gender); we give you P^{ℓ}
- Then we define groups based on 2 covariates; we give you P^{ℓ}
- Then we define groups based on 2 covariates and ask you to construct P^{ℓ} based on marginal distributions

Tackling Problem 1: MRP

Problem 1: Estimate $\hat{y}^{\ell} = \frac{\sum_{j \in \{j \mid A_j = 1, x = \ell\}} Y_j}{|\{j \mid A_j = 1, x = \ell\}|}$ might be really bad

Too few respondents in a group → high variance (1 person might determine entire average)

- Somehow this seems wrong: presumably, the estimate for a group should be very close to that of a "neighboring" group
- "Multi-level regression with post-stratification" (MRP)

Main idea: Train a (Bayesian) regression model to get estimate $\hat{y}^{\ell}$ for each set of covariates. Then, "post-stratify" by weighting $\hat{y}^{\ell}$ by population fraction P^{ℓ}

For groups with many samples, estimate $\hat{y}^{\ell}$ just based on that group; otherwise, based on "neighboring" groups

Parting thoughts on weighting

- Where do the population percentages come from? In political polling, you need to define a universe of “likely voters”
- Methods not covered here: *Inverse Propensity Scoring*, and *Matching*
- Note, can only weight when you observe the covariates for each respondent!
- What if sampling bias is correlated with a feature you don’t observe?
  - Next time!

Parting thoughts

Be purposeful! Does your numeric data capture what you want it to?

Be skeptical! Just because a poll was “random” doesn’t make it good

Unmeasured confounding and quantifying uncertainty

[Extra content, not covered in class]

The challenge

- In the last lecture, weighting helped us deal with *measured* selection bias/differential non-response

  Response rates and political opinions both correlate with educational status;

  (1) Education status can be asked for during the poll
  (2) We can roughly guess at voter distribution by education status
  (3) Then use various *weighting* techniques

- What if response rates & opinions depend on a covariate that we don’t observe, or that we don’t know the population distribution of?
- Very little we can do to recover “point-estimate” of population opinion
- However, we can *quantify the uncertainty* under *assumptions* on how bad the problem is

Setup

- Suppose there is a (binary) covariate u_j that correlates with both the opinion of interest Y_j and whether people respond A_j.
- You don't observe u_j for any individual j
- u is the only unmeasured confounding: A_j is uncorrelated with true opinion Y_j given u_j -- but we don't have u_j
- You have an estimate $\hat{y}$ (raw average of responses)
- Idea: Make assumptions on "how bad" the unmeasured confounding can get to derive uncertainty regions for your estimate of interest.

How to quantify uncertainty

- If we assume like we did on the last slide: “Conditional on what group the respondent belongs to, their opinion does not correlate with whether they respond”
- Then, you can do some math where your error decomposes into the difference between groups in *whether they respond* and *true opinion differences*

$$\hat{y} - \bar{y} \to \left(\tilde{P}^1 - P^1\right)\left(E\left[Y_j \mid u_j = 1\right] - E\left[Y_j \mid u_j = 0\right]\right)$$

More detail: Notation and Insight

- True population fractions of u: $\mathrm{P}^1 = \Pr(u_j = 1), 1 - \mathrm{P}^1 = \Pr(u_j = 0)$
- Response fractions: $\tilde{\mathrm{P}}^\ell = \Pr(u_j = \ell \mid A_j = 1)$
- $\bar{y} \stackrel{\text{def}}{=} E[Y_j] = P^1 E[Y_j \mid u_j = 1] + (1 - P^1) E[Y_j \mid u_j = 0]$
- $\hat{y} \to E[Y_j \mid A_j = 1] = \tilde{\mathrm{P}}^1 E[Y_j \mid u_j = 1, A_j = 1]$
  $+ (1 - \tilde{P}^1) E[Y_j \mid u_j = 0, A_j = 1]$
- Insight:

$$E[Y_j \mid u_j = \ell, A_j = 1] = E[Y_j \mid u_j = \ell]$$

"Conditional on what group the respondent belongs to, their opinion does not correlate with whether they respond" ← We assumed this on last slide!

More detail: Quantifying uncertainty in math

$$\bar{y} = P^1 E[Y_j \mid u_j = 1] + (1 - P^1) E[Y_j \mid u_j = 0]$$
$$\hat{y} \rightarrow \tilde{P}^1 E[Y_j \mid u_j = 1] + (1 - \tilde{P}^1) E[Y_j \mid u_j = 0]$$

Rearrange:

$$\hat{y} \rightarrow \bar{y} + \left(\tilde{P}^1 - P^1\right) E[Y_j \mid u_j = 1] + \left(P^1 - \tilde{P}^1\right) E[Y_j \mid u_j = 0]$$
$$= \bar{y} + \left(\tilde{P}^1 - P^1\right) \left(E[Y_j \mid u_j = 1] - E[Y_j \mid u_j = 0]\right)$$

Then, make assumptions on *whether respond* and *opinion* differences to quantify how far $\hat{y}$ can be from $\bar{y}$

If *either* response fractions or opinions between groups are similar, effect of unmeasured confounding is small!

Unmeasured confounding in ML

- In data science, we often care about *causal inference* (later in semester)
  - "What is the causal effect of going to a private high school on college success?"
  - Problem: In the US, private HS attendance correlated with parents' wealth
- Unmeasured confounding (you might not know parents' wealth) would mess up your *inference* of the relationship in a regression
- You can also quantify unmeasured confounding and range of effects in such cases

Questions?